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TWO-DIMENSIONAL CONVERSE PROBLEMS FOR QUASILINEAR THERMAL CONDUCTIVITY EQUATIONS

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Converse problems on determination of unknown functions which depend on solution of the original problem and the spatial variable are studied.

One-dimensional converse problems involving unknown functions dependent on the solution of the original problem were considered in [1]. In view of the fact that solutions of converse problems are sought within special classes of functions, we will first define those classes.

Definition 1. We will say that a function q(u, x) belongs to the class $\mathfrak{M}_\beta[R_1, R_2]$, if $q(u, x) \in C^{3,2}([R_1, R_2] \times [0, \infty)) \cap C((-\infty, \infty) \times [0, \infty))$ and the following conditions are satisfied: $q_x'(u, x) \leqslant 0$ for $u \geqslant 0$, and for any two functions of the given class their difference $\tilde{q}(u, x) = q_1(u, x) - q_2(u, x)$ satisfies the inequality

$$\|\tilde{q}_x'(u, x)\|_u \leqslant c\|\tilde{q}(u, x)\|_u, \quad \left\|\frac{\partial^{k+m} q(u, x)}{\partial x^k \partial u^m}\right\| \leqslant \beta, \quad k+m \leqslant 2$$

, where c is a fixed constant.

Definition 2. The function $\sigma(u, x) \in \mathfrak{M}_\beta^k[R_1, R_2]$, if $\sigma(u, x) \in C^{3+k,2+k}([R_1, R_2] \times [0, \infty)) \cap C^{k,k}((-\infty, \infty) \times [0, \infty))$, and the conditions $\left\|\frac{\partial^{k+m}\sigma(u, x)}{\partial x^k \partial u^m}\right\| \leqslant \beta,\ k+m \leqslant 2,\ 0 < \nu \leqslant \sigma(u, x) \leqslant \mu$, are valid, and for any two functions of the given class their difference $\tilde{\sigma}(u, x) = \sigma_1(u, x) - \sigma_2(u, x)$ satisfies the inequality $\|\tilde{\sigma}_x'(u, x)\|_u \leqslant c\|\tilde{\sigma}(u, x)\|_u$.

We will now note some facts necessary for the future.

Lemma 1. Let $\varphi(t)$ be a continuous function at $0 \leqslant t \leqslant T$ and

$$\varphi(t) \leqslant \psi(t) + c\int_0^t \varphi(\tau)\left(1 + \frac{1}{\sqrt{t-\tau}}\right)d\tau, \quad 0 \leqslant t \leqslant T$$

, then for any t^*, $0 \leqslant t^* \leqslant T$,

$$\max_{0 \leqslant t \leqslant t^*} |\varphi(t)| \leqslant c_1 \max_{0 \leqslant t \leqslant t^*} |\psi(t)|,$$

where the constant c_1 depends on c and T.

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Lemma 2. Let the functions $\varphi(x) \in H^{2+\alpha}([0, \infty))$, $\psi(t) \in H^{1+\alpha/2}([0, T])$ satisfy the conditions $\varphi(x) > 0$, $\varphi'(x) < 0$, $0 \leqslant x < \infty$, $\psi'(t) \geqslant \alpha_1 > 0$, and $\varphi(x) \in H^{2+\alpha}([0, \infty))$, $\psi(t) \in H^{1+\alpha/2}([0, T])$ $0 \leqslant t \leqslant T$. Then for the classical finite solution of the problem

$$u_t = u_{xx} + q(u, x), \ x_0 < x < \infty, \ 0 < t \leqslant T, \tag{1}$$

$$u|_{t=0} = \varphi(x - x_0), \quad x_0 \leqslant x < \infty, \tag{2}$$

$$u|_{x=x_0} = \psi(t), \quad 0 \leqslant t \leqslant T, \tag{3}$$

the estimate $0 \leqslant u(x, t) \leqslant \psi(t)$, $0 \leqslant t \leqslant T$, $x_0 \leqslant x < \infty$ is valid.

Lemma 3. Within the assumptions of Lemma 1 there exist a point $x^* < \infty$, such that in the region $\{x_0 + x^* \leqslant x < \infty, \ 0 \leqslant t \leqslant T\}$ the inequality $u(x, t) \leqslant \psi(0)$ is satisfied.

Lemma 4. Upon fulfillment of Lemmas 1 and 3 and for $f(t, x_0) \in H^{1+\alpha/2}([0, T])$, $f(t, x_0) \leqslant -\alpha_2 < 0$, in the region $\{x_0 \leqslant x \leqslant x_0 + x^*, \ 0 \leqslant t \leqslant T\}$ for the solution of the problem of Eqs. (1)-(3), the estimate $u_x(x, t) \leqslant -c < 0$ is valid, where the function $f(t, x_0)$ is taken from the condition

$$u_x|_{x=x_0} = f(t, x_0), \ 0 \leqslant t \leqslant T. \tag{4}$$

We will now turn to the direct formulation of the converse problem. We will consider within the region $D(T, x_0) = \{x_0 < x < \infty, \ 0 < t \leqslant T\}$ for Eq. (1) the boundary problem (2), (3) assuming that the additional information of Eq. (4) is known for the finite solution of this problem. Assuming that x_0 varies over the range from zero to infinity, our problem then consists of defining the function $q(u, x)$ from a known function $f(t, x_0)$.

Theorem. Let the functions $\varphi(x)$, $\psi(t)$, $f(t, x_0)$ satisfy the conditions of Lemmas 2 and 4, and let the consistency condition $\varphi(0) = \psi(0)$, $\varphi'(0) = f(0, x_0)$ be satisfied. Then the solution of the converse problem is unique and stable relative to a small change in the function $f(t, x_0)$ in the class of functions $q(u, x) \in \mathfrak{M}_\beta[0, \psi(T)]$, coinciding with each other in the range $0 \leqslant u \leqslant \psi(0)$, $0 \leqslant x < \infty$.

Proof. We will assume that having specified the auxiliary $f_1(t, x_0)$ we have found a solution of the converse problem which we denote by $\{u_1(x, t), q_1(u_1, x)\}$. Specifying another function $f_2(t, x_0)$, we find another solution of the problem of Eqs. (1)-(4), which we denote by $\{u_2(x, t), q_2(u_2, x)\}$, while $q_1(u_1, x) \in \mathfrak{M}_\beta[0, \psi(T)]$, $q_2(u_2, x) \in \mathfrak{M}_\beta[0, \psi(T)]$.

We take $v(x, t) = u_1(x, t) - u_2(x, t)$, $\tilde{q}(u, x) = q_1(u, x) - q_2(u, x)$, $\tilde{f}(t, x_0) = f_1(t, x_0) - f_2(t, x_0)$. Then for v and $\tilde{q}(u_2, x)$ we obtain problem which reduces to an integral equation of the first kind for determination of the function $\tilde{q}(u_2, x)$. With the aid of differentiation and Abel's approach, the equation thus obtained can be reduced to a Volterra equation of the second kind, from which the uniqueness theorem follows and an estimate of stability can be obtained.

Similar results were obtained for equations with an unknown coefficient before the first derivative $u_t = \sigma(u, x) u_{xx}$ or for an equation written in divergent form: $u_t = (\sigma(u, x) u_x)_x$.

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